Package ‘mvmesh’

April 11, 2015

Type Package

Title Multivariate Mesbes and Histograms in Arbitrary Dimensions

Version 1.0

Date 2015-04-10

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Depends R (>= 3.0), rcdd, rgl

Description Define, manipulate and plot meshes on simplices, spheres, balls, and rectangles for use in multivariate statistics. Directional and other multivariate histograms are provided.

License GPL (>= 3)

NeedsCompilation no

Repository CRAN

Date/Publication 2015-04-11 01:19:14

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**Description**

Define, manipulate and plot multivariate meshes/grids in n-dimensional Euclidean space. Multivariate histograms based on these meshes are provided.

**Details**

- **Package:** mvmesh
- **Type:** Package
- **Version:** 1.0
- **Date:** 2015-04-10
- **License:** GPL (>= 3)

A range of multivariate problems require looking at simplices, spheres, balls and rectangular grids in dimension n > 1. Examples are multivariate stable distributions or multivariate extreme value distributions, where a probability distribution is specified by a measure on a sphere or simplex. Also, simulation of generalized spherical laws involves a triangulation of some surface. Quadrature problems on a simplex or sphere require the ability to specify and work with meshes, e.g. packages SphericalCubature and SimplicialCubature. Another application of this is multivariate histograms. For example, directional histograms tablulate the number of points in a sequence of directions, see function `histDirectional`.

A key goal here is that the dimension n is not limited to 2 or 3, but in principle can be arbitrary. Of course, as n increases compute times and required memory will increase quickly. This package uses existing methods from computational geometry that work in arbitrary dimension. Several of these functions were written as prototypes, so getting something to work was the immediate goal, speed was not.

In this documentation we will use the term grid to mean a collection of points, usually approximately evenly spread on a solid or surface. We will use the term mesh to mean both the grid, and the grouping information that tells which points make up the simplices that triangulate the region.

Please let me know if you find any mistakes. I will try to fix bugs promptly.

Constructive comments for improvements are welcome; actually implementing any suggestions will be dependent on time constraints.

This research was supported by an agreement with Cornell University, Operations Research & Information Engineering, under contract W911NF-12-1-0385 from the Army Research Development and Engineering Command.

**Version history:**

- 1.0 (2015-03-01) original package

The remainder of this section describes some of the internal details of the package. It is not needed for the average users.
Points in n-dimensional space are stored in row vectors as is customary in R. All simplices considered in this package are convex. A single convex simplex can be described/stored in two ways:

- A vps x n matrix of (doubles) S; the rows S[1,], S[2,], etc. are the vertices in R^n. The simplex is the convex hull of the vertices. Note: vps stands for ‘vertices per simplex’.
- An nV x n matrix of (doubles) vertices V with rows giving the points in R^n, and an integer vector of length vps called SVI (=Simplex Vertex Indices) that specifies which vertices make up a simplex.

Both of these descriptions have their uses, so the core functions in this package calculate both. Most geometric objects described be a list of simplices. To store all the relevant information needed, the basic functions in this package return an object of class mvmesh. An object of class mvmesh has the following fields, extending the definitions above from a single simplex to a list of simplices:

- type - a string describing the mesh, e.g. "unit simplex" (see table below)
- mvmesh.type - an integer specifying the type of mesh (see table below)
- n - dimension of the space
- m - dimension of the mesh, e.g. the unit sphere in n=3 dimensions is an m=2 dimensional surface. (see table below)
- vps - vertices per simplex, the number of vertices that define a simplex, which must be the same for all simplices in this mesh (see table below)
- S - an (vps x n x nS) array, with S[ , ,k] specifying the vertices of k-th simplex
- V - an (nV x n) matrix giving the distinct vertices in the list of simplices (repeated vertices in S that are on common edges are removed)
- SVI - an integer (vps x nS) matrix which specifies the indices of the vertices that make up the simplices in S. SVI = Simplex Vertex Indices. SVI[ ,k] gives the subscripts in the vertex array V that determine the k-th simplex in S
- other fields are specific to the type of mesh. Generally, they describe the parameters that were used to generate the mesh

<table>
<thead>
<tr>
<th>type</th>
<th>mesh.type</th>
<th>m</th>
<th>vps</th>
</tr>
</thead>
<tbody>
<tr>
<td>unit simplex</td>
<td>1</td>
<td>n-1</td>
<td>n</td>
</tr>
<tr>
<td>solid simplex</td>
<td>2</td>
<td>n</td>
<td>n+1</td>
</tr>
<tr>
<td>unit sphere, edgewise</td>
<td>3</td>
<td>n-1</td>
<td>n</td>
</tr>
<tr>
<td>unit sphere, dyadic</td>
<td>4</td>
<td>n-1</td>
<td>n</td>
</tr>
<tr>
<td>unit ball, edgewise</td>
<td>5</td>
<td>n</td>
<td>n+1</td>
</tr>
<tr>
<td>unit ball, dyadic</td>
<td>6</td>
<td>n</td>
<td>n+1</td>
</tr>
<tr>
<td>rectangular</td>
<td>7</td>
<td>n</td>
<td>2^n</td>
</tr>
<tr>
<td>icosahedron</td>
<td>8</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>polar sphere</td>
<td>9</td>
<td>n-1</td>
<td>2^(n-1)</td>
</tr>
<tr>
<td>polar ball</td>
<td>10</td>
<td>n</td>
<td>2^(n-1) + 1</td>
</tr>
</tbody>
</table>

Currently two generic S3 methods work for objects of class mvmesh: print and plot.

Notes: This package represents points in n dimensional space as double precision numbers. This is
convenient, but has potential problems. For example, determining whether points lie on a line or in a plane may not be possible with floating point arithmetic because coordinates can’t be represented exactly. The computational geometry package rcdd on CRAN gives a way around this by using exact rational arithmetic. This works fine for points on a linear space, but not for points on the unit sphere: (sqrt(2)/2,sqrt(2)/2) is on the unit circle, but cannot be represented exactly as a rational number. So, we use floating point numbers everywhere. When required package rcdd is loaded, it prints out a warning message about double precision numbers and encourages the use of rational arithmetic. I do not know how to suppress this message.

Examples

```r
UnitSimplex( n=2, k=3 )
UnitBall( n=3, k=2 )

## Not run:
plot( SolidSimplex( n=2, k=3 ), col="red" )
title("2d solid simplex")
plot( SolidSimplex( n=3, k=4 ) )
plot( UnitSimplex( n=3,k=4), new.plot=FALSE, col="red", lwd=5 )
title3d("unit simplex and solid simplex in 3d")
rgl.viewpoint( -45, 15 )

plot( UnitSphere( n=3, k=2 ), col="blue")
mesh2 <- AffineTransform( UnitBall( n=3,k=2 ), A=diag(c(1,1,1)), shift=c(3,0,0) )
plot( mesh2, new.plot=FALSE, col="magenta" )
title3d("triangulation of unit sphere and ball in 3d")

demo(mvmesh) # shows a range of meshes
demo(mvhist) # shows a range of multivariate histograms
demo(mvmesh) # miscellaneous examples

## End(Not run)
```

mvhist

Multivariate histograms

Description

Tabulate and plot histograms for data, including directional histograms

Usage

```r
histDirectional( x, k, p=2, plot.type="default", freq=TRUE, positive.only=FALSE, report="summary", ... )
```
mvhist

histRectangular( x, breaks=10, plot.type="default", freq=TRUE, report="summary", ...)
histSimplex( x, S, plot.type="default", freq=TRUE, report="summary", ...)

TallyHrep( x, H, report="summary" )
DrawPillars( S, height, shift=rep(0,0,3), ... )

Arguments

x data in an (n x d) matrix; rows are d-dimensional data vectors
k number of subdivisions
p power of p-norm
freq TRUE for a frequency histogram, FALSE for a relative frequency histogram
breaks specifies the subdivision of the region; see 'breaks' in RectangularMesh
plot.type type of plot, see details below
positive.only If TRUE, look only in the first octant
report level of warning messages; one of "summary", "all", "none".
... Optional arguments to plot
S (vps x d x nS) array of simplices in V representation, see V2Hrep
H array of simplices in H representation, see V2Hrep
height vector of length nS giving the heights of the pillars
shift shift of the pillars, typically (0,0,0) for 2d data or (0,0,z0) for 3d data

Details

Calculate and plot multivariate histograms. In all cases, the bins are simplices computed from some description. Then the number in each simplex is tallied using TallyHrep.

'plot.type' values depend on the type of plot being used. Possible values are:

- "none" - does not show a plot, just return the counts
- "index" - shows a histogram of simplex index number versus count, does not show the geometry, but works in any dimension
- "pillars" - shows a 3D plot with pillars/columns having base the shape of the simplices and height proportional to frequency counts. When the points are 2D, this works for histRectangular and histSimplex; when the points are 3D, this only works for histRectangular
- "counts" - shows frequency counts as a number in the center of each simplex
- "radial" - histDirectional only, shows radial spikes proportional to the counts
- "grayscale" - histDirectional only, shows radial spikes proportional to the counts
- "orthogonal" - histDirectional only, shows radial spikes proportional to the counts
- "default" - type depends on the dimension of the data and type of histogram
Value

A plot is drawn (unless plot.type="none"). A list is returned invisibly, with fields:

- counts - frequency count in each bin
- nrejects - number of x values not in any bin
- nties - number of points in more than one bin (if bins are set up to be non-overlapping, this should only occur on a shared edge between two simplices
- nx - total number of data points in x
- rel.freq - counts/nx
- rel.rejects - nrejects/nx
- mesh - object of type mvmesh, see mvmesh
- plot.type - input value
- report - input value

Examples

# two dimensional, isotropic
x <- matrix( rnorm(8000), ncol=2 )
histDirectional( x, k=1 )
histRectangular( x, breaks=5 )

## Not run:

# three dimensional positive data
x <- matrix( abs(rnorm(9000)), ncol=3 )
histDirectional( x, k=3, positive.only=TRUE, col='blue', lwd=3 )
histRectangular( x, breaks=4 )

demo(mvhist) # shows a range of multivariate histograms

## End(Not run)

mvmesh-geom

Miscellaneous functions for working with multivariate meshes

Description

EdgeSubdivision calculates an equal area/volume subdivision of a simplex. AffineTransformMesh define new mesh by translating all vertices by A Rotate2D and Rotate3D calculate rotation matrices for use by AffineTransform.

Icosahedron returns the vertices of the icosahedron with vertices on the unit sphere
Other functions are internal functions, use at your own risk.
**Usage**

```r
EdgeSubdivision( n, k )
EdgeSubdivisionMulti( V, SVI, k, normalize = FALSE, p = 2)
ConvertBase( m, b, n)
NumVertices( n, k, single = TRUE)
PointCoord( S, color )
SimplexCoord( S, color )
SVIFromColor( S, T )

MatchRow(v, table, first = 1, last = nrow(table))
AffineTransform( mesh, A, shift )
Rotate2D( theta )
Rotate3D( theta )
Icosahedron( )

V2Hrep( S )
H2Vrep( H )
SatisfyHrep( x, Hsingle )
HrepCones( S )
```

**Arguments**

- **v**: a vector of length `n`
- **table**: matrix of size `m3 x n`
- **first**: row to start search
- **last**: row to end search
- **mesh**: object of class "mvmesh"
- **A**: `n x n` matrix
- **shift**: shift vector of length `n`
- **theta**: rotation angle; in 2D, this is a single angle; in 3D is it a vector of length 3, with `theta[i]` giving rotation around `i`-th axis
- **...**: optional parameters to rgl plot commands
- **k**: number of subdivisions
- **n**: dimension of simplex
- **V**: matrix of vertices; each row is a point in `R^n`
- **normalize**: TRUE to normalize vertices to lie on the unit sphere in the `l^p` norm
- **p**: power in the `l^p` norm
- **S**: matrix of size `(vps x n)` specifying the vertices of a single simplex; `S[j,]` is the `j`-th vertex of `S`
- **SVI**: Simplex Vertice Index, see `mvmesh`
- **m**: positive integer to be converted to base `b`
- **b**: positive integer, the base used to express `x`
- **single**: If TRUE, return only one value; if FALSE, return table of values
color matrix, internal matrix used by EdgeSubdivision to subdivide a simplex

T array giving a list of color matrices

H array of simplices in the H-representation, H[,k] is the H-representation for the k-th simplex

x matrix with columns giving the points

Hsingle matrix giving the H-representation of a single simplex

Details

AffineTransform computes a new mesh from a previous one, with each vertex v being replaced by A Rotate3D computes a 3D rotation matrix.

Icosahedron returns the vertices of the icosahedron with vertices on the unit sphere

H2Vrep converts from the half-space (H) representation to the vertex (V) representation of a simplex. V2Hrep converts from the V-representation to the H-representation. It is assumed that all the resulting value are of the same dimension. If this is not the case, an error will occur. To work with such cases, call the function separately for each simplex and save the result in different size objects. The one place where this can occur with mvmesh objects is with a PolarSphere or PolarBall: at the places where polar coordinates are nonunique, vertices will repeat and the H-representation will have fewer constraints than other simplices.

Value

MatchRow returns an integer vector, showing which rows of table match v. If there are no matches, it returns integer(0).

AffineTransform returns an object of class "mvmesh". Rotate2D returns a 2 x 2 rotation matrix, Rotate3D returns a 3 x 3 rotation matrix.

EdgeSubdivision computes an edgewise subdivision of a simplex using the method of Edelsbrunner and Grayson. The algorithm of Concalves, et. al. was implemented in R. It is a coordinate free method. ConvertBase is an internal routine used by the subdivision algorithm. NumVertices is a utility routine to recursively calculate the number of vertices in an edgewise subdivision.

EdgeSubdivMulti is roughly a vectorized version of EdgeSubdivision. It takes a list of simplices, and performs a k-subdivision of each simplex for function UnitSphere and related functions. Since some simplices may share edges, the same vertex can be occur multiple times, so this function goes through the resulting vertices and eliminates repeats. This function is not meant to be called by an end user; it is not guaranteed to be general.

ConvertBase is an internal function that converts a positive integer 'x' to an 'n' digit base 'b' representation. NumVertices is an internal function that computes the number of simplices in an edgewise subdivision (without doing the subdivision). PointCoord is an internal function that computes a single vertex of a simplex. SimplexCoord is an internal function that computes the coordinates of a simplex 'S' given color matrix 'color'. SVIFromColor is an internal function that computes the SVI from a starting simplex 'S' and color array 'T'.

Note that rays and lines are not allowed in V2Hrep; use rcdd function makeH directly to use them.

EdgeSubdivision returns a color matrix, a coordinate free representation of the subdivision. One generally uses UnitSimplex or UnitBall to get a vertex representation of the subdivision.

EdgeSubdivMulti returns a list of class 'mvmesh'
References


Examples

Icosahedron( )
T <- EdgeSubdivision( n=2, k=2 )
T
ConvertBase( 10, 2, 6 ) # note order of digits
NumVertices( n=4, k=8, single=FALSE )
S <- rbind( diag(rep(1,2)), c(0,0) ) # solid simplex in 2D
PointCoord( S, T[,1] )
SimplexCoord( S, T[,1] )
SVIFromColor( S, T )
S1 <- rbind( c(0,0,0), diag( rep(1,3) ) )
S2 <- rbind( c(1,1,1), diag( rep(1,3) ) )
S3 <- rbind( c(1,1,1), c(0,1,0), c(1,0,0), c(1,1,0) )
S <- array( c(S1,S2,S3), dim=c(4,3,3) )
(H1 <- V2Hrep( S ) )
(S4 <- H2Vrep( H1 ) )
(H2 <- HrepCones( UnitSphere(n=2,k=1)$S[,2]) ) # cone between 0 <= y <= x, x >= 0
x <- matrix( rnorm(100), ncol=2 )
(i <- SatisfyHrep( x, H2 ) )
x[i,]
(table <- matrix( c(1:12,1:3 ), ncol=3, byrow=TRUE ))
MatchRow( 1:3, table )

## Not run:
plot( Icosahedron( ), col="green" )
mesh <- SolidSimplex( n=3, k=2 )
plot(mesh, col="blue")
mesh2 <- AffineTransform( mesh, A=Rotate3D( rep(pi/2,3) ), shift=c(1,1,1) )
plot(mesh2, new.plot=FALSE, col="red" )

## End(Not run)
Methods to print and draw mvmesh objects

Description

Print summary of a mesh and plot 2D and 3D simplices. The 2D plot routines use the standard R plots; 3D plot routines use the rgl package.

Usage

## S3 method for class 'mvmesh'
print(x, ...)  

## S3 method for class 'mvmesh'
plot(x, new.plot=TRUE, show.points=FALSE, show.edges=TRUE, show.faces=FALSE, show.labels = FALSE, label.values=NULL, ...)  
DrawSimplex2d(S,label,show.labels,mvmesh.type,show.edges=TRUE,show.faces=FALSE,...)  
DrawSimplex3d(S,label,show.labels,mvmesh.type,show.edges=TRUE,show.faces=FALSE,...)

Arguments

- **x**: an object of class "mvmesh", usually from one of the functions Unitsimplex, SolidSimplex, UnitSphere, UnitBall, RectangularMesh, etc.
- **new.plot**: If TRUE, start a new plot; otherwise add to an existing plot
- **show.points**: If TRUE, show vertices (use cex= to change size)
- **show.edges**: If TRUE, show edges
- **show.faces**: If TRUE, fill in solid faces (only works in certain cases); otherwise show edges
- **show.labels**: If TRUE, an identifying label will be drawn inside each simplex
- **label.values**: values to display if show.label=TRUE; defaults to 1,2,3,...
- **...**: Optional argument to plot functions to set color, alpha, etc.
- **label**: Integer to label current simplex
- **S**: a simplex, an n x m matrix with columns S[1],...,S[m] giving the vertices
- **mvmesh.type**: integer code identifying what type of mesh this is, see the definition of class "mvmesh" in mvmesh.

Details

print will print out summary information about a mesh object
plot will plot a mesh, calling DrawSimplex2d or DrawSimplex3d to plot a each simplex as appropriate for the dimension. These routines are meant to give a basic display; not all rgl capabilities are used.

Value

A plot is drawn, usually nothing is returned
Examples

```r
print(SolidSimplex( n=3, k=2 ) )

## Not run:
plot( SolidSimplex( n=3, k=2 ), col='red' )

## End(Not run)
```

---

**PolarSphere**

*Define a mesh on the unit sphere/ball in n-dimensions determined by a polar coordinates grid.*

---

**Description**

Subdivide the unit ball or sphere into simplices in arbitrary dimensions using a rectangular grid on the polar parameterization of the sphere.

The general n-dimensional polar coordinates to and from rectangular coordinates transformations are provided.

**Usage**

```r
PolarSphere(n, breaks=c(rep(4,n-2),8), p = 2, positive.only = FALSE)
PolarBall( n, breaks=c(rep(4,n-2),8), p=2, positive.only=FALSE )
Rectangular2Polar( x )
Polar2Rectangular( r, theta )
```

**Arguments**

- **n**: Dimension of the space; the Polar sphere is an (n-1) dimensional manifold
- **breaks**: specification of the partition of in the angle space theta. See the definition of 'breaks' in `RectangularMesh`
- **p**: Power used in the l^p norm; p=2 is the Euclidean norm
- **positive.only**: TRUE means restrict to the positive orthant; FALSE gives the full ball
- **r**: a vector of radii of length m.
- **theta**: a (n-1) x m matrix of angles.
- **x**: (n x m) matrix, with column j being the point in n-dimensional space.
PolarSphere computes an approximation to the unit sphere using a rectangular grid in the polar angle space. PolarBall uses a partition of the polar sphere and joins those simplices to the origin to approximately partition the unit ball. LpNorm computes the \( l^p \) norm of each columns of \( x \).

Polar2Rectangular and Rectangular2Polar convert between the polar coordinate representation \((r, \theta_1, \ldots, \theta_{n-1})\) and the rectangular coordinates \((x_1, \ldots, x_n)\).

\( n \) dimensional polar coordinates are given by the following:
- rectangular \( x=(x_1, \ldots, x_n) \) corresponds to polar \((r, \theta_1, \ldots, \theta_{n-1})\) by
  \[
  \begin{align*}
  x[1] &= r \cos(\theta[1]) \\
  x[2] &= r \sin(\theta[1]) \cos(\theta[2]) \\
  x[3] &= r \sin(\theta[1]) \sin(\theta[2]) \cos(\theta[3]) \\
  \vdots \\
  x[n-1] &= r \sin(\theta[1]) \sin(\theta[2]) \cdots \sin(\theta[n-2]) \cos(\theta[n-1]) \\
  x[n] &= r \sin(\theta[1]) \sin(\theta[2]) \cdots \sin(\theta[n-2]) \sin(\theta[n-1])
  \end{align*}
  \]
- Here \( \theta_1, \ldots, \theta_{n-2} \) in \([0, \pi)\), and \( \theta_{n-1} \) in \([0, 2\pi)\). This is the parameterization described in the Wikipedia webpage for "n-sphere". Note that this is NOT a 1-1 transformation: when \( \theta_1=0 \), it follows that \( x[2]=x[3]=\ldots=x[n]=0 \). This is analagous to all longitude lines going through the north pole in standard 3d spherical coordinates.
- For multivariate integration, the Jacobian of the above tranformation is \( J(\theta) = r^{n-1} \prod(\sin(\theta[1:(n-2)]))^{(n-2):1} \); note that \( \theta_{n-1} \) does not appear in the Jacobian.

Value

PolarSphere and PolarBall return an object of class "mvmesh" as described in mvmesh. Polar2Rectangular returns an \((n \times m)\) matrix of rectangular coordinates. Rectangular2Polar returns a list with fields:
- \( r \) a vector of length \( m \) containing the radii
- \( \theta \) an \((n \times m)\) matrix of angles

Examples

PolarSphere(n=3, breaks=4)
PolarBall(n=3, breaks=4)

(x <- matrix(1:10, ncol=2))
(a <- Rectangular2Polar(x))
Polar2Rectangular(a$r, a$theta)

(x <- matrix(1:12, ncol=4))
(a <- Rectangular2Polar(x))
Polar2Rectangular(a$r, a$theta)

## Not run:
plot(PolarSphere(n=2, breaks=8))
plot(PolarBall(n=2, breaks=8))

plot(PolarSphere(n=3, breaks=c(4,8)))
plot(PolarBall(n=3, breaks=c(4,8)))
RectangularMesh

Subdivide a hyperrectangle with a standard grid

Description

EdgeSubdivision implements the

Usage

RectangularMesh( a, b, breaks=5, silent=FALSE )
NextMultiIndex( i, n )

Arguments

a vector specifying the "lower left" vertex of the rectangle
b vector specifying the "upper right" vertex of the rectangle
breaks a specification of the subdivision scheme. See details below.
silent indicates whether or not to warn the caller if the subdivision determined by
'breaks' covers the whole hyperrectangle [a,b].
i integer vector
n integer vector

Details

RectangularMesh computes an rectangular mesh on the hyperrectangle [a,b] = [a[1],b[1]] x [a[2],b[2]]
x ... x [a[n],b[n]]. It is similar to the function mesh in CRAN package plot3D, but works for dimension d=2,3,4,...

'breaks' determines how each component is divided, it is motivated by the argument breaks in
hist. If 'breaks' is a vector of length n, then breaks[i] gives the number of evenly sized bins
in coordinate i, spread out over the range [a[i],b[i]]. If 'breaks' is a single number m, then each
component is subdivided into that many bins, i.e. this is equivalent to breaks=rep(m,n). Thus the
default breaks=6 subdivides each coordinate into 6 bins. Finally, if a more complicated subdivision
is desired, 'breaks' can a list with n fields. breaks[[i]] should be a vector of dividing points for
coordinate i. See the example below. In this last case, where the bin boundaries are explicitly
defined, 'a' and 'b' are not used (other than a possible warning if the specified bins do not cover the
rectangle given by 'a' and 'b').

Value

An object of class "mvmesh" as described in mvmesh.
Examples

RectangularMesh( a=c(1,3), b=c(2,7), breaks=2 )
RectangularMesh( a=c(1,3), b=c(2,7), breaks=c(4,10) )
RectangularMesh( a=c(1,3), b=c(2,7),
  breaks=list( seq(1,3,by=0.25), seq(2,7,by=1) ) )

## Not run:
plot( RectangularMesh( a=c(1,3), b=c(2,7), breaks=3 ), show.labels=TRUE )
plot( RectangularMesh( a=c(1,3), b=c(2,7), breaks=c(4,10) ), show.labels=TRUE )
plot( RectangularMesh( a=c(1,3), b=c(2,7),
  breaks=list( seq(1,3,by=0.25), seq(2,7,by=1) ) ), show.labels=TRUE )
plot( RectangularMesh( a=c(1,3), b=c(2,7), breaks=3 ), show.labels=TRUE,
  label.values=letters[1:9], col='green' )
plot( RectangularMesh( a=c(1,3,0), b=c(6,7,6), breaks=3 ), show.labels=TRUE, col='blue' )

## End(Not run)

UnitSimplex  Define a mesh on the unit simplex or the canonical simplex

Description

Defines an equal area/volume subdivision of the unit simplex and the canonical simplex in \( \mathbb{R}^n \). The
unit simplex is the \((n-1)\) dimensional simplex with vertices \((1,0,0,...,0), (0,1,0,...,0), ...,(0,0,0,...,1)\),
i.e. all \( x \geq 0 \) with \( \text{sum}(x)=1 \).

The solid simplex is the \( n \) dimensional simplex with vertices \((1,0,0,...,0), (0,1,0,...,0), ...,(0,0,0,...,1)\),
and \((0,0,...,0)\), i.e. all \( x \geq 0 \) with \( \text{sum}(x) \leq 1 \).

Usage

\texttt{UnitSimplex}(n, k)
\texttt{SolidSimplex}( n, k )

Arguments

\( n \)  
dimension of the space

\( k \)  
number of subdivisions

Details

\texttt{EdgeSubdivision} is called to do a \( k \)-subdivision of each edge, and then that output is converted to
a matrix of vertices.

Value

an object of class "mvmesh" as described in \texttt{mvmesh}. 
UnitSphere

Examples

UnitSimplex( n=2, k=3 )
SolidSimplex( n=2, k=3 )

UnitSimplex( n=3, k=2 )
SolidSimplex( n=3, k=2 )

UnitSimplex( n=5, k=4 )
SolidSimplex( n=5, k=4 )

## Not run:
plot( UnitSimplex( n=2, k=3 ) )
plot( SolidSimplex( n=2, k=3 ) )

plot( UnitSimplex( n=3, k=2 ) )
plot( SolidSimplex( n=3, k=2 ) )

## End(Not run)

UnitSphere

Define a mesh on a unit ball in n-dimensions

Description

Subdivide the unit ball or sphere into approximately equal simplices in arbitrary dimensions.

Usage

UnitSphere(n, k, method = "dyadic", p = 2, positive.only = FALSE)
UnitSphereEdgewise(n, k, p, positive.only)
UnitSphereDyadic(n, k, start = "diamond", p, positive.only)
UnitBall( n, k, method="dyadic", p=2, positive.only=FALSE )
LpNorm(x, p)

Arguments

n Dimension of the space; the unit sphere is an (n-1) dimensional manifold
k Number of subdivisions
method "dyadic" or "edgewise": the former recursively subdivides the sphere to get a more uniform grid; the latter uses a faster method using one edgewise subdivision.
p Power used in the l^p norm; p=2 is the Euclidean norm
positive.only TRUE means restrict to the positive orthant; FALSE gives the full ball
start starting shape: "diamond" or "icosahedron"
x Matrix of points in n-dimensions; each column is a point in R^n
UnitSphere computes a hyperspherical triangle approximation to the unit sphere. It calls either UnitSphereDyadic or UnitSphereEdgewise based on 'method'. Both work by subdividing the first octant, and then rotating that subdivision around to other octants. Note that 'k' has a different meaning for the different methods. When method="dyadic", k specifies the number of dyadic subdivisions. When method="edgewise", k specifies the number of subdivisions as in UnitSimplex, which is then projected outward to the unit sphere. So when n=2, a dyadic subdivision with k=2 will result in 16 edges, whereas an edgewise subdivion with k=2 results in 8 edges.

UnitBall computes an approximate simplicial approximation to the unit ball. Specifically, it generates cones with one vertex at the origin and the other vertices on the surface of the unit sphere; these later vertices are from UnitSphere. If k is large, these cones will be very narrow/thin.

Value

an object of class "mvmesh" as described in mvmesh.

Examples

UnitSphere( n=2, k=2, method="edgewise", positive.only=TRUE )
UnitSphere( n=2, k=2, method="edgewise" )

UnitSphere( n=3, k=2, method="edgewise", positive.only=TRUE )
UnitSphere( n=3, k=2, method="edgewise" )

UnitBall( n=2, k=2, method="edgewise", positive.only=TRUE )
UnitBall( n=2, k=2, method="edgewise" )

UnitSphere( n=3, k=2, method="dyadic", positive.only=TRUE )
UnitSphere( n=3, k=2, method="dyadic" )

UnitBall( n=3, k=2, method="dyadic", positive.only=TRUE )
UnitBall( n=3, k=2, method="dyadic" )

UnitSphere( n=3, k=2 )
UnitBall( n=3, k=2 )

x <- c(3,-1,2)
LpNorm( x, p=2 )

## Not run:
plot( UnitSphere( n=3, k=2 ), show.label=TRUE )
plot( UnitBall( n=3, k=2 ) )

## End(Not run)
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