Algorithmic Issues in Real Time Ads Sale

S. Muthu Muthukrishnan
Rutgers Univ
Online Ad Markets

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- Online Ad Markets:
  - Where are the ads shown?
  - What are the creatives?
  - What is the targeting language?
  - What is a charging event?
  - What is under the hood: prediction, allocation, bidding, reporting and attribution, budget management, optimization and campaign goals.
Display Ads

Common practice:

Ads and prices determined by offline negotiations.
Alternative: Ad Exchange (AdX)

Online Ad Allocation

- Advertisers $A_i$ sets targeting and budget $B_i$ ahead of time.
- Each arriving impression $j$ has a set of $(i, j)$ of advertisers $A_i$ who fit.

$$\begin{align*}
\text{max} & \quad \sum_{(i,j) \in E} y_{ij} \\
\text{s.t.} & \quad \sum_i y_{ij} \leq 1 \\
& \quad \sum_{j|(i,j) \in E} y_{ij} \leq 1 \\
& \quad y_{ij} = \{0, 1\}
\end{align*}$$

$$\begin{align*}
\text{max} & \quad \sum_{(i,j) \in E} b_{ij} y_{ij} \\
\text{s.t.} & \quad \sum_i y_{ij} \leq 1 \\
& \quad \sum_{j|(i,j) \in E} b_{ij} y_{ij} \leq B_i \\
& \quad y_{ij} = \{0, 1\}, \ b_{ij} \ll B_i
\end{align*}$$

Table: Matching

Table: AdWords
AdWords

- Offline, NP Hard. $1 + \epsilon$ approx. LP rounding.

- Online $1 - 1/e$ approx. worst case. The MSVV Algorithm:
  - For advertiser $i$ and for the arriving query $q$ define the scaled bid $\hat{b}_{iq} = b_{iq} \psi(f_i)$ where $f_i$ is the fraction of unspent budget of $i$, and $\psi(x) = 1 - e^{-x}$. For each arriving query we allocate it to the advertiser with the highest scaled bid.

- Online, iid or random order. $1 - \epsilon$ approx when OPT is somewhat larger than any edge weight.
Ad Allocation: Display Ads

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$$\begin{align*}
\text{max} & \quad \sum_{(i,j) \in E} w_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_i x_{ij} \leq 1 \\
& \quad \sum_{j|(i,j) \in E} x_{ij} \leq N_i \\
& \quad x_{ij} = \{0, 1\}
\end{align*}$$

No bounded competitive ratio possible ($N_1 = 1$, $w_1 = 100$ and $w_2$ is 10000 or 1).
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Display Ads with Free Disposal

- Impressions satisfy Free Disposal property in Economics: optimize the weight of $N_i$ largest weight edges to $i$. 

\[ \max_{P(i; j)} \sum_{E} w_{ij} x_{ij} \quad \text{s.t.} \quad P_i x_{ij} \leq 1 \quad \text{and} \quad P_j x_{ij} \leq \sum_{E} x_{ij} \quad \text{for} \quad i + j \leq w_{ij} x_{ij}. \]

Algorithm: Maintain feasible solutions to primal and dual online. Assign $j$ to $\text{argmax}_i w_{ij}$. Update $i$ as exponential weighted average of $N_i$ largest weighted impressions.

Claim: $(1 - 1/e)$ approximation as $N_i \to 1$. Used operationally in DoubleClick.
Display Ads with Free Disposal

- Impressions satisfy Free Disposal property in Economics: optimize the weight of $N_i$ largest weight edges to $i$.

- Technical approach: $x_{ij}$ denotes whether impression $j$ is one of the $N_i$ most valuable impressions assigned to $i$.

\[
\begin{align*}
\max & \quad \sum_{(i,j) \in E} w_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_i x_{ij} \leq 1 \\
& \quad \sum_{j \mid (i,j) \in E} x_{ij} \leq N_i
\end{align*}
\]

\[
\begin{align*}
\min & \quad \sum_i N_i \beta_i + \sum_j \alpha_j \\
\text{s.t.} & \quad \beta_i + \alpha_j \geq w_{ij} \\
& \quad x_{ij}, \beta_i, \alpha_j \geq 0
\end{align*}
\]

Claim: ($1 = e$) approximation as $N_i! \to 1$. Used operationally in DoubleClick.
Impressions satisfy free disposal property in Economics: optimize the weight of $N_i$ largest weight edges to $i$.

Technical approach: $x_{ij}$ denotes whether impression $j$ is one of the $N_i$ most valuable impressions assigned to $i$.

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\begin{align*}
\text{max} & \quad \sum_{(i,j) \in E} w_{ij} x_{ij} \\
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& \quad \sum_{j \mid (i,j) \in E} x_{ij} \leq N_i
\end{align*}
\]

Algorithm: Maintain feasible solutions to primal and dual online. Assign $j$ to $\arg\max_i \{w_{ij} - \beta_i\}$. Update $\beta_i$ as exponential weighted average of $N_i$ largest weighted impressions.

Claim: $(1 - 1/e)$ approximation as $N_i \to \infty$. Used operationally in DoubleClick.
Ad Allocation with Intermediaries

- Display ad markets use ad exchanges which call brokers or ad networks which in turn represent adv.
- Publishers use multiple intermediate buy networks including ad exchanges.
- Challenge: intermediaries have capacity bottlenecks.
Ad Allocation with Call Out

- Prob that intermediary $I_i$ bids $k$ on impression $j$ is $p_{ijk}$.
- $I_i$ has rate $\rho_i$ of impressions it can handle.
- **Problem:** For each impression $j$, call out to set $S_j$ of networks and satisfy rate constraints. Maximize $\sum_j \text{max bid from } S_j$.
- **Claim:** Assuming impressions are drawn from unknown distribution, there is an online algorithm with $1 - 1/e - \varepsilon$ approximation. Primal dual approach, except solves a simple LP per impression.
- Used in InMobi. Arises in Yahoo/Bing contract.
Ad Allocation Summary

- Ad allocation: Matching and AdWords.
- Ad allocation with Free Disposal, for display ads.
- Ad allocation with selective callout, for ad markets with intermediaries.
- Ad allocation with myriad variants, and other optimization problems in online ad markets. Can add the price/floor component.
Input is a set of keywords and a budget.
- For each keyword, \((\text{clicks}, \text{cost})\) pair.
- Assume same auction all day, same competitors, bids.
- Maximize the number of clicks obtained.
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For each keyword, \((\text{clicks}, \text{cost})\) pair.

Assume same auction all day, same competitors, bids.

Maximize the number of clicks obtained.

Result: [FMPS07]
Let \(C\) be the number of clicks obtained by an Omniscent bidder.
There exists a bid \(b\) such that \(\text{clicks}(\text{uniform}(b)) \geq C/2\), where \(\text{uniform}(b)\) represents bidding \(b\) on all keywords.
Budget Optimization Contd

**Figure:** Omniscent Bidder

**Figure:** Approx bidding
Major Directions

- Bidding and budget optimization for a variety of metrics.
- Not covered in this talk: the learning aspects, game theory and auction aspects.
Major Research Directions

- **Bulk Performance.** Allocate ads in such a way that advertisers see some “smooth” performance over time. Need a “dual” theory that controls supply. Eg., Yelp, Zillow.
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- **Packages.** In vertical ad markets, parties can make side deals for traffic or form packages. Design a market.
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- **Packages.** In vertical ad markets, parties can make side deals for traffic or form packages. Design a market.

- **Micromarket Structure.** Advertisers cluster according to keywords, contexts, etc. Develop an economic theory for identifying these clusters.